

## Problem 3 : Lexicographic Permutations

We will show the solution for the millionth permutation and items given by  $\{a, b, c, d, e, f, g, h, i, j\}$ .

- $N = 1000000$ .
- $I = \{a, b, c, d, e, f, g, h, i, j\}$ .
- Note that  $9! < N < 10!$ .
- Note that we can permute  $l$  items in  $l!$  ways. So there are  $9!$  permutations which have an  $a$  as first item and  $9!$  permutations which have a  $b$  as first item, etcetera.

To determine the first letter of the permutation, we calculate

$$(1000000 - 1) \bmod(9!) = 274239, (999999 - (999999 \bmod(9!)) / 9! = 2$$

Number remaining: 274240, Letters remaining:  $\{a, b, d, e, f, g, h, i, j\}$

$$274239 \bmod(8!) = 32319, (274239 - (274239 \bmod(8!)) / 8! = 6$$

Number remaining: 32319, indices remaining:  $\{a, b, d, e, f, g, i, j\}$

$$32319 \bmod(7!) = 2079, (32319 - (32319 \bmod(7!)) / 7! = 6$$

Number remaining: 2079, indices remaining:  $\{a, b, d, e, f, g, j\}$ :

$\vdots$

In the end, we have the permutation *chidjbfega*.